

Inference at *
of proof for Lemma can-apply-fun-exp:

$$\begin{array}{l} \vdash \forall A:\text{Type}, f:(A \rightarrow (A + \text{Top})), n:\mathbb{N}, y:A. \\ (\uparrow \text{can-apply}(f^{\wedge} n; y)) \Rightarrow (\forall m:\mathbb{N}. (m \leq n) \Rightarrow (\uparrow \text{can-apply}(f^{\wedge} m; y))) \\ \text{by (Unfold 'can-apply' (} 0 \cdot \text{))} \\ \text{CollapseTHEN } ((\text{InductionOnNat} \\ \text{CollapseTHEN (Auto}\cdot\text{)}\cdot\text{)}) \end{array}$$

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CollapseTHEN ((CaseNat 0 ‘m’)
CollapseTHEN (((Try ((RepUR “
  p-fun-exp p-compose p-id“ ( 0).)
CollapseTHEN (Trivial.)).))
CollapseTHEN ((Subst’
   $m \sim ((m - 1)+1) ( 0).$ )
CollapseTHEN (((Try ((Complete (Auto’)).)).))
CollapseTHEN ((
  RWO ”p-fun-exp-add” 0)
CollapseTHEN ((Auto.)
CollapseTHEN ((MoveToConcl (-4))

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CollapseTHEN ((Subst' n ~ ((n - 1)+1) (0).)
CollapseTHEN (((Try ((Complete (Auto')).
)))
CollapseTHEN ((RWO "p-fun-exp-add" 0)
CollapseTHENA (Auto.).).).).).).
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1:

1. $A : \text{Type}$
 2. $f : A \rightarrow (A + \text{Top})$
 3. $n : \mathbb{Z}$
 4. $0 < n$
 5. $\forall y:A. (\uparrow \text{isl}(f^n - 1(y))) \Rightarrow (\forall m:\mathbb{N}. (m \leq (n - 1)) \Rightarrow (\uparrow \text{isl}(f^m(y))))$
 6. $y : A$
 7. $m : \mathbb{N}$
 8. $m \leq n$
 9. $\neg(m = 0)$
- $$\vdash (\uparrow \text{isl}(f^n - 1 \circ f^{-1}(y))) \Rightarrow (\uparrow \text{isl}(f^m - 1 \circ f^{-1}(y)))$$